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An index of loss aversion

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Abstract

To a considerable extent, risk aversion as it is commonly observed is caused by loss aversion. Several indexes of loss aversion have been proposed in the literature. The one proposed in this paper leads to a clear decomposition of risk attitude into three distinct components: basic utility, probability weighting, and loss aversion. The index is independent of the unit of payment. The main theorem shows how the indexes of different decision makers can be compared through observed choices.

JEL classification: D81; C60

Keywords: Loss aversion; Prospect theory; Rank-dependence; Risk aversion

1. Introduction

Given the abundant evidence of the empirical failures of expected utility, the development of tractable alternative theories for decision under risk is desirable [55,57]. A popular alternative theory is Quiggin's [44] rank-dependent utility. Tversky and Kahneman's [58] new version of prospect theory generalizes Quiggin's theory by adding loss aversion as a new component of risk attitude. Thus, new prospect theory combines the mathematical elegance of Quiggin's theory with the empirical realism of Kahneman and Tversky's [31] original prospect theory.

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Loss aversion reflects the observed behavior that agents are more sensitive to losses than to gains, resulting in a utility function that is steeper for losses than for gains. The phenomenon is empirically well-established and has been used in many applications [32]. However, little theory has been developed so far. Quantifications and parametrizations have been chosen on an ad hoc basis. This paper is the first to propose a behaviorally founded index of loss aversion to govern the exchange rate between gain and loss utility units. This proposal leads to a clear decomposition of risk attitude into three distinct components: basic utility, probability weighting, and loss aversion. We hope that this simple decomposition will contribute in making prospect theory more tractable.

We will see that constant relative risk averse (CRRA, i.e. log-power) utility functions, commonly used in finance and macroeconomics, encounter difficulties when modeling loss aversion. Constant absolute risk averse (CARA, i.e. linear-exponential) utility performs better in this regard. This result suggests that the development of new parametric families of utilities may be desirable.

Although we will formulate our results for decision under risk, they can also be applied to welfare theory, where loss aversion reflects the special sensitivity of consumers to decreases in income. Sign-dependence of welfare transfers, which assigns more importance to the reduction of decreases of income than to the enhancement of increases of income, provides a promising way to generalize existing welfare evaluation models.

The paper is structured as follows. First, definitions are set out in Section 2. Then, our proposed index of loss aversion is developed in Section 3. Section 4 gives a comparative preference foundation by means of Yaari's [63] acceptance sets. Section 5 presents empirical findings, and discusses the role of loss aversion in Rabin's [45] paradox. Section 6 provides conceptual arguments for our proposal, and compares it with other proposals put forward in the literature. Finally, Section 7 presents an analysis of our index for parametric utility families. Proofs are in the Appendix.

2. Prospect theory

Outcomes are monetary, with $\mathbb{R}$ the *outcome set*. We assume that the agent perceives one outcome as the *reference point*, and the other outcomes as changes with respect to this reference point. The reference point often is the status quo. By rescaling the outcomes, we may assume that 0 is the reference point. Then, *gains* are positive amounts and *losses* are negative amounts. A *prospect*, denoted by $P = (p_1, x_1; \ldots; p_n, x_n)$, assigns a nonnegative probability p_i to each outcome x_i, where $\sum_{i=1}^{n} p_i = 1$. Throughout, the outcomes in P are ordered from best to worst, with $x_1 \geqslant \cdots \geqslant x_k \geqslant 0 > x_{k+1} \geqslant \cdots \geqslant x_n$ for some $0 \leqslant k \leqslant n$. L denotes the set of all prospects. Each outcome x is identified with the riskless prospect $(1, x)$. The preferences of an agent over prospects are denoted by $\succcurlyeq$, with indifference denoted by $\sim$.

We next define the new version of prospect theory [58]. A prospect is decomposed into its gain part, where all losses are replaced by the reference point 0, and its loss part, where all gains are replaced by 0. Different rank-dependent formulas are then applied to the gain- and loss-part, and, finally, these two formulas are added. Tversky and Kahneman [58] axiomatized the theory for uncertainty. For risk, the context of this paper, the theory was axiomatized by Chateauneuf and Wakker [16].

Formally, *prospect theory* holds if an evaluation function PT *represents preferences* (i.e. $P \succcurlyeq Q \Leftrightarrow PT(P) \geqslant PT(Q)$ for all prospects P, Q), where PT is defined as follows:

(i) There exists a *utility function* $U : \mathbb{R} \to \mathbb{R}$, which is continuous and strictly increasing with $U(0) = 0$.
(ii) There exist *weighting functions* $w^+, w^- : [0, 1] \to [0, 1]$, which are continuous and strictly increasing with $w^+(0) = w^-(0) = 0$ and $w^+(1) = w^-(1) = 1$.
(iii) $PT(p_1, x_1; \ldots; p_n, x_n) = \sum_{i=1}^n \pi_i U(x_i)$, with $\pi_i = w^+(p_1 + \cdots + p_i) - w^+(p_1 + \cdots + p_{i-1})$ for $i \leqslant k$ (rank-dependent utility for gains with respect to w^+) and $\pi_i = w^-(p_i + \cdots + p_n) - w^-(p_{i+1} + \cdots + p_n)$ for $i > k$ (rank-dependent utility for losses with respect to the dual of w^-).[1]

3. An index of loss aversion

We assume that there exists a basic utility function u that reflects the intrinsic value of outcomes for the agent. Because of the perception of a reference point, however, subjects process losses differently from gains. The observable utility U is a composition of a loss aversion index $\lambda > 0$, reflecting the different processing of gains and losses, and the *basic utility* u. Formally,

$$U(x) = \begin{cases} u(x) & \text{if } x \geqslant 0 \\ \lambda u(x) & \text{if } x < 0. \end{cases}$$

Typically, people pay more attention to losses, λ exceeds 1, and U is steeper for losses than for gains.

Within our model that, like most theoretical studies of prospect theory, is restricted to one fixed reference point, it is impossible to separate u and λ through observed choices without specifying a scaling convention for the gain–loss exchange rate of u. This nonidentifiability of parameters has led to many misunderstandings in the literature. We hope that this paper will help to clarify these misunderstandings, which is why we restricted its scope as we did.

Tversky and Kahneman [58], who used the above decomposition, assumed that $u(x) = x^\alpha$ for gains and $u(x) = -|x|^\beta$ for losses. This amounts to the implicit scaling convention that $u(1) = -u(-1) = 1$, implying $\lambda = \frac{-U(-1)}{U(1)}$ (see Section 6). This scaling convention depends on the unit of payment. We will propose another scaling convention, one that is independent of the unit of payment.

Fig. 1 depicts a typical U. The utility function U has a kink at 0 and is smooth everywhere else. It is plausible that the kink is caused by loss aversion, and does not reflect an intrinsic value of outcomes. That is, it is plausible that the basic utility function u is smooth (differentiable) at 0. We, therefore, define $\lambda = \frac{U'_\uparrow(0)}{U'_\downarrow(0)}$ as the *loss aversion index*. $U'_\uparrow(0)$ denotes the left, and $U'_\downarrow(0)$ the right derivative of U at 0. We assume that $U'_\uparrow(0)$ and $U'_\downarrow(0)$ exist, and are positive and finite. The assumption of smoothness of u at 0 serves, so to speak, as our scaling convention for the gain–loss exchange rate.

[1] In particular, $\pi_1 = w^+(p_1)$ if $k \geqslant 1$ and $\pi_n = w^-(p_n)$ if $k < n$.

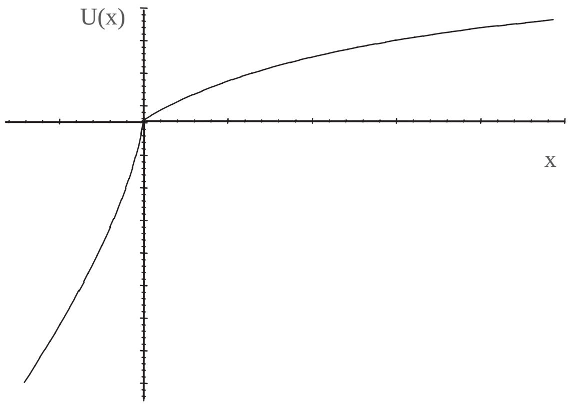

Fig. 1.

It has been suggested before that the kink at 0 reflects loss aversion. For instance, Kahneman [30] writes: “The core idea of prospect theory [is]—that the value function is kinked at the reference point and loss averse—…” (p. 1457). The ratio $\frac{U'_\uparrow(0)}{U'_\downarrow(0)}$ was suggested informally as a measure of loss aversion by Benartzi and Thaler [8, p. 74 l. 5–6]. Our paper formalizes these suggestions. The index λ is invariant under changes of scale of U, and is, therefore, well-defined. It is, moreover, invariant under scale transformations of the outcomes, i.e. it is independent of the unit of payment. Further discussions and justifications of our proposal are given in the following sections, and summarized in the conclusion.

4. Comparative loss aversion

Comparisons of loss aversion can be naturally formulated through Yaari’s [63] acceptance sets, leading to a preference foundation for comparative loss aversion. Our preference condition will imply that weighting functions and utility functions are the same for different agents, which entails a considerable restriction. Similar restrictions, however, applied to the first results on utility and, subsequently, probability weighting, as new components of risk attitude.

The classical results on utility did not consider loss aversion and probability weighting, and thus implicitly assumed that these are the same for all agents [4,5,9,43,48,63 with the same subjective probabilities for different agents]. The first results on probability weighting either assumed linear utility [64,49 with respect to probabilistic mixtures], or did not disentangle probability weighting and utility curvature [14,15,17]. None of the references just mentioned considered loss aversion, which, therefore, is also the same for all agents. Schmidt and Zank [52] characterized strong risk aversion through a joint condition on utility curvature, probability weighting, and our index of loss aversion without, however, disentangling these components. Their result further suggests that our index is a natural component of risk attitude.

Assume two agents, 1 and 2, whose preferences over L, denoted by $\succcurlyeq_1$ and $\succcurlyeq_2$, can be modeled by prospect theory with utility functions U_1, U_2, underlying basic utility functions u_1, u_2, loss aversion indexes λ_1, λ_2, and weighting functions $w_1^+, w_1^-, w_2^+, w_2^-$, respectively. We denote the reference point of each agent by 0, where this can refer to different absolute levels of wealth for the two agents.

L^+ denotes the set of prospects with no loss outcomes, and L^- the set of prospects with no gain outcomes. A prospect is *mixed* if it is contained neither in L^+ nor in L^-, so that it yields both gains and losses with positive probability. For an outcome x, we define $A_1(x) = \{P \in L \mid P \succcurlyeq_1 x\}$ to be agent 1's *acceptance set*, i.e. the set of prospects that the agent prefers to receiving the sure outcome x. The *gain acceptance set* $A_1^+(x) = A_1(x) \cap L^+$ restricts attention to prospects with no loss outcomes. Similarly, the *loss acceptance set* is $A_1^-(x) = A_1(x) \cap L^-$. For $x > 0$, the sets $A_1^+(-x) = L^+$ and $A_1^-(x) = \emptyset$, are empty. Agent 2's acceptance sets $A_2(x)$, $A_2^+(x)$, and $A_2^-(x)$ are defined similarly.

Assume that agent 2's acceptance sets are contained in those of agent 1. This means that agent 2 is more risk averse than agent 1 in Yaari's sense. As will be discussed in Section 6, empirical studies suggest that loss aversion is a major factor in risk aversion. It can therefore be expected that differences in observed risk attitudes of different agents are, to a considerable extent, caused by differences in loss aversion, which is observed only in mixed prospects. This paper characterizes the extreme, but not implausible, case in which no significant differences are found for gain- or loss-prospects, but differences do occur for mixed gambles [6, p. 17]. That is, the gain and loss acceptance sets of the two agents are the same, but for mixed gambles the acceptance sets of agent 1 are larger than those of agent 2. The following theorem shows that this special case of Yaari's preference condition can be characterized in terms of the loss aversion index introduced in Section 3, while implying the same u and w. Observation 2 will demonstrate that Yaari's condition can also be characterized in terms of some other indexes.

Theorem 1. *Assume that the preferences of agents* 1 *and* 2 *can be modeled through prospect theory, such that* $U'_{\uparrow 1}(0)$, $U'_{\downarrow 1}(0)$, $U'_{\uparrow 2}(0)$, *and* $U'_{\downarrow 2}(0)$ *are positive and finite. Then the following Statements* (i) *and* (ii) *are equivalent.*

(i) *The following three conditions hold*:
 (a) $w_2^+ = w_1^+$ *and* $w_2^- = w_1^-$;
 (b) $u_1 = \sigma u_2$ *for some* $\sigma > 0$ (*i.e.* u_1 *and* u_2 *are strategically equivalent*);
 (c) $\lambda_2 \geqslant \lambda_1$.

(ii) *The following two conditions hold*:
 (a) $A_2^+(x) = A_1^+(x)$ *and* $A_2^-(-x) = A_1^-(-x)$ *for all* $x \geqslant 0$;
 (b) $A_2(x) \subset A_1(x)$ *for all* $x \in \mathbb{R}$.

In Theorem 1(i), the conditions (b) and (c) are equivalent to the equality

$$\sigma U_2(x) = \begin{cases} U_1(x) & x \geqslant 0 \\ \gamma U_1(x) & x < 0 \end{cases} \text{ for some } \sigma > 0, \gamma \geqslant 1.$$

The characterizing preference condition can equally well be reformulated in terms of certainty equivalents instead of acceptance sets. That is, Statement (ii) in the theorem can be replaced by

- For all P in L^+ and in L^-, the certainty equivalents of the two agents coincide;
- For all P in L, the certainty equivalent of agent 2 does not exceed that of agent 1.

Under rank-dependent utility and prospect theory, the implications of Yaari's famous definition of comparative risk aversion by means of acceptance sets are as yet unknown. Under expected utility, Yaari's condition characterizes concave transformations of utility. The restriction of Yaari's condition in Theorem 1 also characterizes concave transformations of utility, but of a special kind: with a kink at 0 and linear otherwise. Compared with Yaari's characterization, we have characterized a more restrictive preference condition in a more general model. The restrictive condition precisely captures an important new empirical phenomenon, i.e. comparative loss aversion.

A way to axiomatically disentangle all components of risk attitude remains a topic for future research. It would require isolating loss aversion without imposing identical acceptance sets over the pure gain and the pure loss domain.

5. Empirical findings and suggestions

Empirical studies have suggested that loss aversion is a major factor in observed risk aversion [30, Section III], [32,37,41,47]. It is remarkable that no theoretical foundation exists as yet for such an important empirical concept.

Rabin [45] and Rabin and Thaler [46] provided further evidence for the dependence of decisions on reference points. They assumed an agent who will always turn down a 50–50 gamble of losing \$10 or gaining \$11, independently of the agent's initial wealth, and they derived the following anomaly: under expected utility, this person would have to reject any bet with a 50% risk of losing \$100, no matter how high the potential gains would be. Such an extreme degree of risk aversion is not realistic. Rabin and Thaler suggested that loss aversion, with initial wealth as the reference point, provides a better explanation for the observed preferences than does concave utility.

The separation of the three components, basic utility, probability weighting, and loss aversion, is crucial if variations in the reference point are considered. Shalev [54] considered such variations in game theory, and Bleichrodt et al. [11] used Shalev's model in utility elicitation. A crucial assumption in these models is that basic utility is the same for different reference points. We can then observe the loss aversion index λ by comparing the kink of utility U at a point when it is the reference point with the kink of U at that same point when another point is the reference point. A scaling convention need no longer be invoked (work in progress). Schmidt [50] and Sugden [56] gave general preference axiomatizations for models with varying reference points.

We have introduced utility, probability weighting, and loss aversion as logically independent factors of risk attitude. Their empirical (in)dependence is more intricate. We believe that basic utility is central in normative decisions, and can have empirical meaning independent of the other factors and prior to risk, possibly reflecting preference intensities. However, such interpretations are controversial, with debates dating back to the ordinal revolution of the 1930s. We do not believe that probability weighting is normative, but consider it a psychological factor. No independent empirical implications have been suggested as yet, although underlying psychological factors have been discussed [23,24,60,62]. We, likewise, consider loss aversion to be a psychological factor. It governs the exchange rate between gain and loss utility units, which also affects large stakes, *given a utility function*. Loss aversion will not have direct empirical meaning independent of utility. Under some plausible assumptions about utility, however, loss aversion does have direct empirical meaning, as is explained next.

We expect that the common assumption of linear utility on small domains will be approximately verified on $[0, m]$ and $[-m, 0]$ for moderate positive m, say a month's salary. The marginal utility on the second interval will, however, continue to exceed the marginal utility on the first to a significant degree. The interval $[-m, m]$ will, also for very small m, continue to exhibit considerable deviations from linearity. Therefore, prospects $(.5, x; .5, -x)$, with $0 < x \leqslant m$, will still exhibit considerable risk aversion. We think that the resulting concavity of capital U primarily reflects loss aversion (a kink at 0) rather than smooth concave utility, the classical explanation. Prospects $(.5, R + x; .5, R - x)$ will exhibit considerably less risk aversion if $|R| > m \geqslant x$ (so that they are nonmixed) than if $R = 0$ (so that they are mixed). Empirical support for the latter claim can be found in [26,27]. Both kinds of prospects are similarly affected by probability weighting, and the difference in risk aversion between them is due to loss aversion. In this way, direct estimations of loss aversion can be obtained. Our views on utility agree with those of Rabin and Thaler. First-order risk aversion, discussed mostly for rank-dependent utility [53], may be driven by loss aversion to a great extent.

6. Alternative definitions of loss aversion

Alternative scaling conventions for the gain–loss exchange rate of u can be considered, leading to different definitions of loss aversion. For example, a $\tau > 0$ can be chosen and $-u(-\tau) = u(\tau)\,(= U(\tau))$ can be set, resulting in $\lambda = \frac{-U(-\tau)}{U(\tau)}$. Such different conventions will imply that u is nondifferentiable at 0. Tversky and Kahneman's [58] approach is a special case thereof, with $\tau = 1$ as implicit convention. The same implicit convention was used within the expected utility framework [21,28], in an alternative preference foundation of PT [39], and for life duration as outcome [10]. Our definition $\lambda = \frac{U'_{\uparrow}(0)}{U'_{\downarrow}(0)}$ can be considered the limiting case of τ approaching 0. We chose our definition because it does not depend on the unit of payment, so that it is the same across different countries, and does not require readjustment after inflation or a change of currency, as happened in Europe in 2002.

With utility given, the aforementioned indexes all differ from our index by a positive constant. This implies the following observation, suggested to us by a referee.

Observation 2. *Theorem 1 holds not only for our loss aversion index, but also for any loss aversion index $-U(-\tau)/U(\tau)$ for fixed $\tau > 0$.*

Decompositions of an overall utility function into an underlying basic utility and additional psychological factors have been considered before [11,18, the additional factor being equity, 33, the additional factor being fairness, 38, the additional factor being disappointment, 54]. Sugden [56] considered an alternative decomposition of utility. He assumed no probability weighting, and maximized the expectation of $\varphi(u(x)-u(r))$ with r the reference outcome (that need not be riskless), u a satisfaction function, and φ an evaluation function. This decomposition is reminiscent of a decomposition that became popular in decision analysis in the 1980s [19,34], where the expectation of $\varphi(u(x))$ is maximized, with u a riskless value function and φ an additional component reflecting risk attitude. In this decomposition, there is no reference dependence. Sugden's φ comprises both such additional risk attitude and loss aversion. If u, the reference-independent component in Sugden's model, is taken as the normative component of utility, similar to our basic utility, then his transformation φ could be an additional psychological component that not only comprises loss aversion but also other factors such as numerical sensitivity. Such factors, while ignored in this study and in [11], were investigated empirically in [35].

The loss aversion index λ is of a psychological nature. It is affected by strategically irrelevant perceptions of the reference point, such as those underlying the known discrepancies between willingness to pay and willingness to accept [7]. In some situations it may be deemed desirable to let the basic utility u have a kink at 0, if losses bring genuine extra inconveniences not matched by corresponding gains. Whether such a change of marginal utility around 0 is drastic but still smooth, or abrupt, is not an empirical question, because we do not observe stakes smaller than pennies. It is a question of pragmatic modeling. For consumers who typically receive a fixed amount of money each month and spend it continuously, we think that an abrupt change of marginal utility precisely at the current reference point is implausible [3, p. 432], [56, p. 186]. If there are, however, genuine empirical or pragmatic reasons for an intrinsic kink of utility at some point, and if this point happens to be the reference point, then we think that this kink should be incorporated in the basic utility function and not in loss aversion.

In an interesting study, Huber et al. [29] specified the true preference system of a principal, and asked participants ("agents") to represent the principal in decisions. Remarkably, there still was considerable loss aversion, in deviation from the true preferences. The authors write: "In many settings, one cannot tell whether loss aversion is a bias or merely a reflection of the fact that losses have more emotional impact than gains of equal magnitude. In our choice and rating tasks, however, we found clear evidence that agents motivated to accurately represent the preferences of others gave more weight to negative outcomes than is appropriate" (p. 88).

The scaling convention that we chose to define loss aversion seems to be plausible, but does not cover all concepts of loss aversion advanced in the literature. As these concepts have varied from one context to another, there can, unfortunately, be no one definition that optimally meets all objectives. Examples can be devised in which our definition is counter to some terminologies used before. For example, assume that U is differentiable at 0 so that λ is 1, U is the identity for positive outcomes, and U is very concave for negative outcomes. Then $-U(-y)$ exceeds $U(y)$ for all $y > 0$, a condition which has sometimes been interpreted as loss aversion [31,40], [56, Footnote 10]. In our terminology, the loss aversion index is 1, suggesting no loss aversion. Instead, we say that U is more concave for losses than for gains. Such discrepancies between our definition and others in the literature are not empirically plausible.

Besides the definition of loss aversion of Kahneman and Tversky [31] and others, just mentioned, other definitions of loss aversion have been proposed. Wakker and Tversky [61] defined loss aversion as the requirement that $U'(-x) \geqslant U'(x)$ for all positive x and provided a preference foundation. A comparative extension, and empirical test, of the condition are in [50,51]. Bleichrodt and Miyamoto [10] characterized the condition for the health domain. Neilson [42, Theorem 2] also considered and characterized a weaker condition, requiring that $U(x)/x \geqslant U(y)/y$ for all $x < 0 < y$. Sugden [56, Definition 14] considered the same condition for his gain-/loss evaluation function φ. Bowman et al. [12] and Breiter et al. [13] imposed a stronger condition, namely $U'(x) \geqslant U'(y)$ for all $x < 0 < y$, which was characterized by Neilson [42, Theorem 3]. All these alternative definitions imply both our definition of loss aversion and that of Kahneman and Tversky [31]. The alternative definitions have in common that loss aversion cannot be separated from utility curvature.

Benartzi and Thaler [8] related loss aversion to the local behavior of utility near 0, as this paper does. In this approach, loss aversion is a third component of risk attitude, separate from probability weighting and basic utility and, thereby, from the curvature of utility for gains and losses. We feel that the kink at 0 is more naturally modeled as a separate conceptual component than as part of (basic) utility. The convenience of the resulting three-component decomposition of risk attitude motivated our proposal of the factor λ. Whether λ is called an index of loss aversion or otherwise is, in the end, a terminological issue.

7. Implications for parametric families of utility

The cases of $U'_\uparrow(x)$ and $U'_\downarrow(x)$ equal to 0 or ∞ have been excluded in our definition of loss aversion. In empirical studies, however, constant relative risk aversion is commonly assumed, with utility functions of the form

$$U(x) = \begin{cases} x^r & \text{if } x \geqslant 0 \\ -s(-x)^t & \text{if } x < 0 \end{cases}$$

[28,58]. These functions have extreme (0 or ∞) derivatives at 0 whenever the power is not 1. This point complicates our definition of loss aversion.[2] Tversky and Kahneman [58] found $r = t = 0.88$. In such an exceptional case, with $r = t$, $\frac{U'_\uparrow(0)}{U'_\downarrow(0)}$ can be set equal to s, so that our definition $\lambda = \frac{U'_\uparrow(0)}{U'_\downarrow(0)} (= s)$ may still be meaningful. If $t < r$, then our loss aversion index is infinite, independently of s, which is undesirably extreme. In this case, no basic utility function can be defined.

The most common case of power utility is $r < t$. More precisely, utility is concave for gains, convex for losses [1,22,25,58], and closer to linear for losses than for gains $(0 < r < t < 1)$ [1, p. 1506], [20,36]. Then our loss aversion index is 0, independently of s and, again, no basic utility function can be defined.

[2] This complication was called to our attention by Donkers, personal communication.

In the cases described above, nondegenerate loss aversion can be defined for the other indexes in/above Observation 2. However, then the resulting basic utility is not smooth at 0, and we feel that such loss aversion indexes and basic utilities are ad hoc.

Rabin [45] pointed out that CRRA utilities, while predominantly used in macroeconomics when large stakes are relevant, should not be used on domains with both large stakes and small stakes near 0 [p. 1287]. Our analysis suggests new modeling problems for CRRA utility for mixed prospects. There is another, empirical, problem for CRRA utility for mixed prospects. If $t < r$ then, no matter how small s is, we have $U(x) > -U(-x)$ for sufficiently large $x > 0$, contrary to the empirical findings. If $t > r$ then, no matter how large s is, $U(x)$ exceeds $-U(-x)$ for all x in a sufficiently small neighborhood of 0, again contrary to the common empirical findings. Thus, unless $r = t$, CRRA utility always entails the existence of $U(x) > -U(-x)$, contrary to the empirical findings. The above discussion shows that, for mixed prospects with outcome 0 and degenerate derivatives in the center of the domain, there are several modeling problems for CRRA utility.

We next demonstrate that CARA, i.e. exponential, utility does not encounter the problems at 0 described in the preceding paragraphs. Take

$$u(x) = \begin{cases} \frac{1-e^{-\mu x}}{\mu} & \text{for all } x \geqslant 0 \\ \frac{e^{\nu x}-1}{\nu} & \text{for all } x < 0 \end{cases}$$

as basic utility, and define:

$$U(x) = \begin{cases} \frac{1-e^{-\mu x}}{\mu} & \text{for all } x \geqslant 0 \\ \lambda(\frac{e^{\nu x}-1}{\nu}) & \text{for all } x < 0. \end{cases}$$

The division by μ and ν is a normalization which ensures that the left and right derivatives of the basic utility are 1 at 0. For μ or ν equal to 0, the functions are to be taken as linear. In this family, μ controls the concavity of utility for gains, ν the convexity of utility for losses, and λ the loss aversion. For $\lambda \geqslant 1$ and $0 < \nu < \mu$, this family exhibits some desirable features regarding loss aversion discussed above. We have $-U(-x) > U(x)$ for all $x > 0$ and, even stronger, $U'(-x) = \lambda e^{-\nu x} > e^{-\mu x} = U'(x)$ for all $x > 0$. The latter inequality is the strong form of loss aversion of Wakker and Tversky [61], which implies our condition. Pratt's [43] measure of concavity $A(x) = -U''(x)/U'(x)$ satisfies $0 < -A(-x) = \nu < \mu = A(x)$ for all $x > 0$, showing that U is convex for losses, concave for gains, and closer to linear for losses than for gains, again in agreement with the empirical findings.

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Appendix: proof of Theorem 1

(i) $\rightarrow$ (ii): Let (i.a–c) hold. We have to show that (ii.a–b) hold, of which only (ii.b) is elaborated, because (ii.a) is trivial. Let $x \in \mathbb{R}$ and $P \in A_2(x)$, i.e. $PT_2(P) \geqslant U_2(x)$. Let $P = (p_1, x_1; \ldots; p_n, x_n)$ with $x_1 \geqslant \cdots \geqslant x_k \geqslant 0 > x_{k+1} \geqslant \cdots \geqslant x_n$ for some $0 \leqslant k \leqslant n$. Define $P^+ = (p_1, x_1; \ldots; p_k, x_k; 1 - p_1 - \cdots - p_k, 0)$ and $P^- = (1 - p_{k+1} - \cdots - p_n, 0; p_{k+1}, x_{k+1}; \ldots; p_n, x_n)$. We may, and will, assume $\sigma = 1$. Then $PT_2(P) = PT_2(P^+) + PT_2(P^-) = PT_1(P^+) + \frac{\lambda_2}{\lambda_1} PT_1(P^-) \leqslant PT_1(P)$. If $x \geqslant 0$ then $U_2(x) = U_1(x)$, and, therefore, $U_1(x) \leqslant PT_1(P)$. If $x \leqslant 0$ then $U_1(x) = \frac{\lambda_1}{\lambda_2} U_2(x) \leqslant \frac{\lambda_1}{\lambda_2} PT_2(P) = \frac{\lambda_1}{\lambda_2} PT_2(P^+) + \frac{\lambda_1}{\lambda_2} PT_2(P^-) = \frac{\lambda_1}{\lambda_2} PT_1(P^+) + PT_1(P^-) \leqslant PT_1(P)$. Hence, $P \in A_1(x)$ and, therefore, $A_2(x) \subset A_1(x)$.

(ii) $\rightarrow$ (i): Let (ii.a–b) hold. We have to show that (i.a–c) hold. U_2 and U_1 are continuous and strictly increasing, and $\succcurlyeq_2$ and $\succcurlyeq_1$ coincide on L^+ and L^-. Standard uniqueness results [59, Theorem 12] imply that $w_2^+(p) = w_1^+(p)$ and $w_2^-(p) = w_1^-(p)$ for all $p \in [0, 1]$. Furthermore, $U_2(x) = \alpha U_1(x) + \beta$ for all $x \in \mathbb{R}^+$, and $U_2(x) = \gamma U_1(x) + \delta$ for all $x \in \mathbb{R}^-$ with $\alpha, \beta, \gamma, \delta \in \mathbb{R}$ and $\alpha, \gamma > 0$. We have $U_2(0) = U_1(0) = 0$ so that $\beta = \delta = 0$, and may normalize $U_2(1) = U_1(1) = 1$ (i.e. $\sigma = 1$ and, now, $\alpha = 1$). It then follows that

$$U_2(x) = \begin{cases} U_1(x) & x \geqslant 0 \\ \gamma U_1(x) & x \leqslant 0. \end{cases}$$

It is left to show that $\gamma \geqslant 1$. Let $P = (p, 1; 1 - p, -1)$ be such that $PT_2(P) < 0$ and $0 < p < 1$. Because U_2 is continuous, there exists a $y \in \mathbb{R}^-$ with $U_2(y) = PT_2(P)$, implying $P \in A_2(y)$. We have, with the inequality implied by (ii.b), $w_1^+(p)U_1(1) + w_1^-(1-p)\gamma U_1(-1) = w_2^+(p)U_2(1) + w_2^-(1-p)U_2(-1) = PT_2(P) = U_2(y) = \gamma U_1(y) \leqslant \gamma PT_1(P) = \gamma w_1^+(p)U_1(1) + \gamma w_1^-(1-p)U_1(-1)$. Because $w_1^+(p) > 0$ and $U_1(1) > 0$, this implies $\gamma \geqslant 1$. $\square$

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